1. Explain the main stages of DE optimization algorithm.

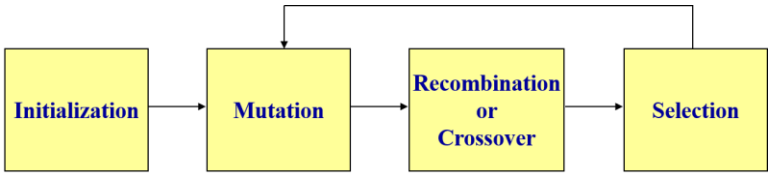


Figure 1: Basic Stages of DE

Initialization

- All parameter vectors in a population are randomly *initialized*.
- Define upper and lower bounds for each parameter:

$$x_j^L \leq x_{j,i,1} \leq x_j^U$$

- Randomly select the initial parameter values uniformly on the intervals

$$[\, x_j^L \,,\, x_j^U \,]$$

- Suggestion to choose random values between high bound and low bound

$$x_{j,i,1} = x_j^L + \text{rand}*(x_j^U - x_j^L)$$

Mutation

- Mutation, recombination and selection will be run for each of the *NP* parameter vectors of a population.
- For a given parameter vector $x_{i,G}$ (called target vector) randomly select three vectors $x_{r1,G}$, $x_{r2,G}$ and $x_{r3,G}$ such that the indexes *i*, *r1*, *r2* and *r3* are distinct integers $\in \{1, 2, \ldots, NP\}$.
- Add the weighted difference of the two vectors $x_{r2,G}$, $x_{r3,G}$ to the base vector $x_{r1,G}$.

$$v_{i,G+1} = x_{r1,G} + F.(x_{r2,G} - x_{r3,G})$$

- The mutation factor *F* is a constant from [0 , 2] which controls the amplification of the differential variation $(x_{r2,G} - x_{r3,G})$.
- $v_{i,G+1}$ is called the *mutant* vector or *donor* vector

Crossover

- Crossover incorporates successful solutions from the previous generation.
- The trial vector $u_{i,G+1}$ is developed from the elements of the target vector $x_{i,G}$ and the elements of the mutant vector $v_{i,G+1}$.

$$u_{j,i,G+1} = \begin{cases} v_{j,i,G+1} & \text{if } (randb(j) \leq CR) \text{ or } j = Irand \\ x_{j,i,G} & \text{if } (randb(j) > CR) \text{ and } j \neq Irand \end{cases},$$
$$i = 1,2,...,NP \; ; \; j = 1,2,...,D.$$

- *randb(j)* is the j[th] evaluation of a uniform random number generator with out come $\in [0, 1]$.
- *CR* is called "the crossover rate" and it is a constant $\in [0, 1]$ which has to be determined by the user.
- *Irand* is a random integer from [1, 2, ...,D] which ensures that the trial vector $u_{i,G+1}$ gets at least one parameter from $v_{i,G+1}$.

Selection

- The target vector $x_{i,G}$ is compared with the trial vector $u_{i,G+1}$ and the one with the lowest cost function value is chosen to the next generation

$$x_{i,G+1} = \begin{cases} u_{i,G+1}, \text{ if } \left(f(u_{i,G+1}) \leq f(x_{i,G})\right) \\ x_{i,G}, \text{ "otherwise"} \end{cases}$$

- Mutation, recombination and selection continue until some stopping criterion is reached

2. Why the population size of DE should be ≥ 4?

Because its required in mutation process For a given parameter vector $x_{i,G}$ target vector select another three vectors

3. Explain the labels of the following variants of DE:

➢ DE/rand/1/bin

- **DE**: Differential Evolution
- **rand**: Base vector for mutation is chosen randomly

- **1**: one difference vector is used to construct the donor
- **bin**: crossover is binomial
- $v_{i,G+1} = x_{r1,G} + F.(x_{r2,G} - x_{r3,G})$

➢ **DE/rand/2/bin**

- **DE**: Differential Evolution
- **rand**: Base vector for mutation is chosen randomly
- **2**: two difference vectors is used to construct the donor
- **bin**: crossover is binomial
- $v_{i,G+1} = x_{r1,G} + F.(x_{r2,G} - x_{r3,G}) + F.(x_{r4,G} - x_{r5,G})$

➢ **DE/best/1/bin**

- **DE**: Differential Evolution
- **best**: The base vector is the vector with the best fitness in the current population.
- **1**: one difference vector is used to construct the donor
- **bin**: crossover is binomial
- $v_{i,G+1} = x_{best,G} + F.(x_{r1,G} - x_{r2,G})$

➢ **DE/best/2/bin**

- **DE**: Differential Evolution
- **best**: The base vector is the vector with the best fitness in the current population.
- **2**: two difference vectors is used to construct the donor
- **bin**: crossover is binomial
- $v_{i,G+1} = x_{best,G} + F.(x_{r1,G} - x_{r2,G}) + F.(x_{r3,G} - x_{r4,G})$

4. Explain the mutation process to get the mutant (or donor) vectors, assume the mutation factor F= 0.5 and the current population contains the following vectors (NP =4):

$$\mathbf{P}=\begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{bmatrix}$$

Where: $x1 = [\ 1\ 1\ ]$ $x2 = [\ 2\ 4\ ]$ $x3 = [\ 0\ -2\ ]$ $x4 = [\ 3\ 2\ ]$

Target vector $x_{1,G} = x1 = [\ 1\ 1\ ]$
$X_{r1,G} = x2 = [\ 2\ \ 4\ ]$
$X_{r2,G} = x3 = [\ 0\ \ -2\ ]$
$X_{r3,G} = x4 = [\ 3\ \ 2\ ]$
$v_{1,G+1} = x_{r1,G} + F.(x_{r2,G} - x_{r3,G})$
$= [2\ \ 4] + 0.5\ ([0\ -2] - [3\ 2]) = [2\ 4] + 0.5[-3\ \ -4] = [2\ \ 4] + [-1.5\ \ -2] = [0.5\ \ 2]$

Target vector $x_{2,G} = x2 = [\ 2\ 4\ ]$
$X_{r1,G} = x1 = [\ 1\ \ 1\ ]$
$X_{r2,G} = x3 = [\ 0\ -2\ ]$
$X_{r3,G} = x4 = [\ 3\ \ 2\ ]$
$V_{2,G+1} = x_{r1,G} + F.(x_{r2,G} - x_{r3,G})$
$= [1\ \ 1] + 0.5\ ([0\ -2] - [3\ 2]) = [1\ 1] + 0.5[-3\ \ -4] = [1\ \ 1] + [-1.5\ \ -2] = [-0.5\ \ -1]$

Target vector $x_{3,G} = x3 = [\ 0\ \ -2\ ]$
$X_{r1,G} = x1 = [\ 1\ \ 1\ ]$
$X_{r2,G} = x2 = [\ 2\ \ 4\ ]$
$X_{r3,G} = x4 = [\ 3\ \ 2\ ]$
$V_{3,G+1} = x_{r1,G} + F.(x_{r2,G} - x_{r3,G})$
$= [1\ \ 1] + 0.5\ ([2\ \ \ 4] - [3\ 2]) = [1\ \ 1] + 0.5[-1\ \ \ 2] = [1\ \ 1] + [-0.5\ \ 1] = [0.5\ \ 2]$

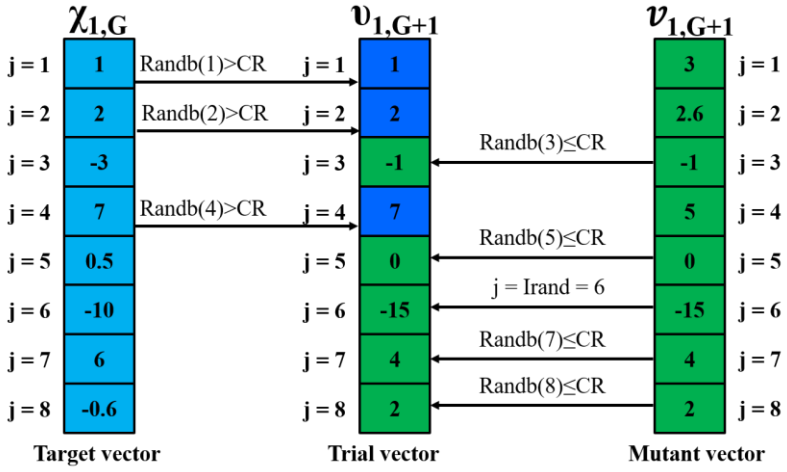
Target vector $x_{4,G} = x4 = [\ 3\ \ 2]$
$X_{r1,G} = x1 = [\ 1\ \ 1\ ]$
$X_{r2,G} = x2 = [\ 2\ \ 4\ ]$
$X_{r3,G} = x3 = [\ 0\ \ -2\ ]$
$V_{4,G+1} = x_{r1,G} + F.(x_{r2,G} - x_{r3,G})$
$= [1\ \ 1] + 0.5\ ([2\ \ 4] - [0\ -2]) = [1\ 1] + 0.5[2\ \ 6] = [1\ \ 1] + [1\ \ 3] = [2\ \ 4]$

5. Explain the crossover process to get the trial vector $u_{1,G+1}$ from the following two vectors:

$$x_{1,G} = \begin{bmatrix} \mathbf{1} \\ \mathbf{2} \\ \mathbf{-3} \\ \mathbf{7} \\ \mathbf{0.5} \\ \mathbf{-10} \\ \mathbf{6} \\ \mathbf{-.6} \end{bmatrix} \qquad v_{1,G+1} = \begin{bmatrix} \mathbf{3} \\ \mathbf{2.6} \\ \mathbf{-1} \\ \mathbf{5} \\ \mathbf{0} \\ \mathbf{-15} \\ \mathbf{4} \\ \mathbf{-2} \end{bmatrix}$$

Assume the random integer value Irand = 6, the crossover rate CR = 0.7 and the uniform random numbers generator randb is

randb = [0.8 0.9 0.1 0.9 0.6 0.9 0.2 0.5]



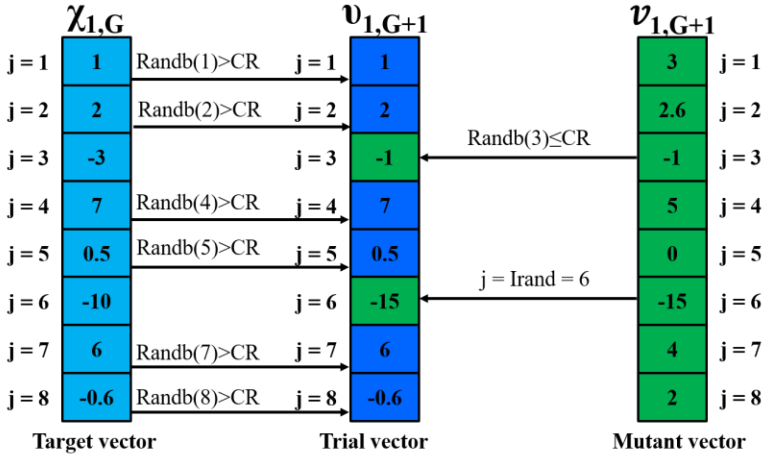
$$u_{1,G+1} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 7 \\ 0 \\ -15 \\ 4 \\ 2 \end{bmatrix}$$

6. What are the dimensionality of the optimization problem in Q(4) and Q(5)?

For Q(4): D =2

For Q(5): D = 8

7. Repeat Q(5) with CR = 0.1 and explain how CR values control the change of the trial vector values $u_{i,G+1}$ than the target vector $x_{i,G}$.



$$u_{1,G+1} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 7 \\ 0.5 \\ -15 \\ 6 \\ -0.6 \end{bmatrix}$$

- A small value of *CR* (0.1) in Q7 leads to most trial vector components being chosen from the target vector.
- Large values of *CR* (0.7) in Q5 leads to most trial vector components being chosen from the mutant vector.

8. What are the control parameters for the DE algorithm?

The parameters that control the performance of DE are three:

(1) The population size ***NP***

(2) The mutation factor ***F***

(3) The crossover rate ***CR***

9. Explain the effects of the control parameters CR and F on the performance of DE.

- **The mutation factor *F*** is relevant to the convergence speed as it is responsible for the step size that interferes in the formation of the mutant vector.
- Small values of *F* will lead to premature convergence
- *F* > 1 will try to take large steps, leading to slow convergence.
- A good initial choice of *F* is 0.5 and the effective range usually lies in [0.4, 1].

- **The crossover rate *CR*** controls the number of changes in parameters of a population member.
- A small value of *CR* ('strong' crossover e.g. 0 or 0.1) leads to most changes occurring along one dimension or a small subset of dimensions, and this is useful for separable functions.
- Large values of *CR* (near 1) lead to most components being chosen from the mutant vector and this is suitable for non-separable functions.